

Using Tiered Lessons in Mathematics

THE MOVEMENT TOWARD INCLUSION has impacted classrooms by requiring teachers to respond to a broader range of academic needs. How can we possibly reach all the students in our classrooms when they are academically diverse, have special needs, and are ESL learners? One answer to this question is to differentiate instruction.

The tenets of differentiated instruction support both the Equity Principle and the Teaching Principle of *Principles and Standards for School Mathematics* (NCTM 2000). These Principles support selecting and adapting content and curriculum to

meet the interests, abilities, and learning styles of our students, to recognize our students' diversity, and to encourage them to reach their full potential in mathematics.

What Is Differentiation?

ALTHOUGH DIFFERENTIATED INSTRUCTION is not new, the differentiation movement has taken center stage as a way to meet the needs of all students in the classroom. It is an organized yet flexible way of proactively adjusting teaching and learning to meet students where they are and help all students achieve maximum growth as learners (Tomlinson 1999). Instruction may be differentiated in content, process, or product according to the students' readiness, interests, or learning style. By *content*, we mean the material that is being presented. *Process* activities help students practice or make sense out of the content; whereas *product* refers to the outcome of the lesson or unit, such as a test, project, or paper. *Readiness* refers to prior knowledge and a student's current skills and proficiency with



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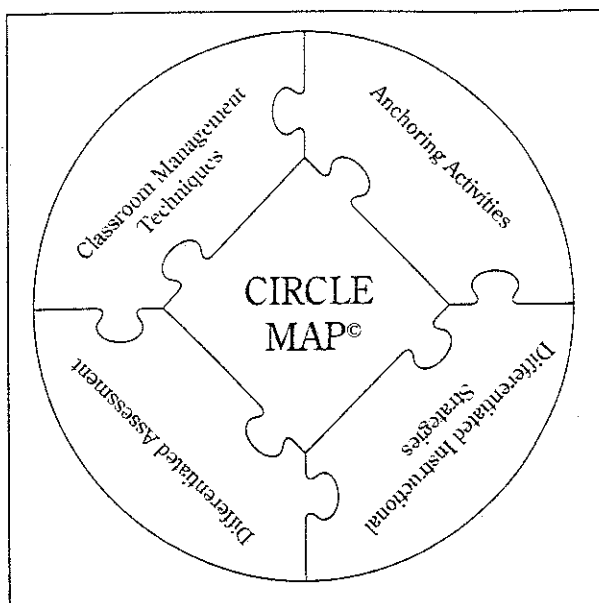


Fig. 1 An illustration of the CIRCLE MAP, which stands for **Creating an Integrated Response for Challenging Learners Equitably: A Model by Adams and Pierce**

the material presented in the lesson. A student's *interests* may be assessed with an interest inventory for the particular topic being studied or by an individual conversation with the student. Many teachers use multiple intelligences as ways to characterize *learning styles* (Armstrong 1994; Gardner 1993; Martin 1996). A variety of instruments can determine a student's learning style. (Two good interactive instruments may be found at www.ldpride.net.)

Having worked with preservice and in-service teachers at a university laboratory school over the last decade on implementing differentiated instruction, we noticed several commonalities among the successful teachers. As a result, we have developed a model: Creating an Integrated Response for Challenging Learners Equitably: A Model by Adams and Pierce (CIRCLE MAP). The CIRCLE MAP, shown in **figure 1**, is appropriate at any grade level and content area. It interweaves four elements—classroom management techniques, anchoring activities, differentiated instruction strategies, and differentiated assessment. With this model, *all* students engage in meaningful, respectful work at a moderately challenging level. For differentiation of instruction to occur successfully, the teacher moves from being the dispenser of knowledge to a facilitator of knowledge; the student moves from being a consumer of knowledge to a producer.

The Classroom Management Component

THE CLASSROOM MANAGEMENT COMPONENT of the CIRCLE MAP must include rules for working in a variety of configurations. Flexible grouping

arrangements, such as pairs, triads, or quads, as well as whole-group and small-group instruction, create opportunities to meet individual needs. A flexible use of time allows lessons to proceed to their natural conclusion rather than be carried out in set blocks of time.

Since you can only work with one group or individual at a time, we have found two critical rules that prevent confusion and preserve sanity. The first rule, "Use six-inch voices," means that students should modulate their speaking level so that their voices can only be heard six inches away. The second rule is "Ask three before me." If students need help completing a task or come to a stumbling block, they should ask three other students first. If these students cannot answer the question, the student has permission to ask the teacher.

The Anchoring Component

STUDENTS ARE GIVEN ANCHORING ACTIVITIES, also called *sponge activities*, to use when they are waiting for help before going any further or have completed their work and are waiting to begin the next lesson. They are also used at the beginning of the class period to get students ready to work. These activities include relevant extension and enrichment work, individual assignments, practice with specific skills, and teacher-selected and student-selected activities. Anchoring activities replace "I'm done" thinking with "What's next?" Students know that it is always safe to work on an anchoring activity—they may get redirected later by the teacher, but they will not get in trouble when doing an anchoring activity.

The Differentiated Instruction Strategies Component

A WIDE VARIETY OF STRATEGIES CAN BE USED to differentiate instruction (see Gregory and Chapman 2002; Heacox 2002; Smutny, Walker, and Meckstroth 1997; Tomlinson 1999; and Winebrenner 1992 for a discussion of other strategies). However, the focus of this article is on a specific strategy—*tiered lessons*. Tomlinson (1999) describes tiered lessons as "the meat and potatoes of differentiated instruction." A tiered lesson is a differentiation strategy that addresses a particular standard, key concept, and generalization, but allows several pathways for students to arrive at an understanding of these components, based on the students' *readiness, interests, or learning styles*. A lesson tiered by readiness level implies that the teacher has a good understanding of the students' ability levels with respect to the lesson and has

designed the tiers to meet those needs. Think of a wedding cake with tiers of varying sizes. Many examples of lessons tiered in readiness have three tiers: below grade level, at grade level, and above grade level. However, no rule states that there may be only three tiers. The number of tiers depends on the range of ability levels in the specific classroom. Tiers are formed based on our assessment of our students' abilities to handle the material particular to a lesson. Students are regrouped when we use a different tiering strategy. When the lesson is tiered by interest or learning style, we are looking at student characteristics other than ability level. For example, the lesson might be tiered to focus on three learning styles: auditory, visual, and kinesthetic. Students would then be placed in the tier that best matches their learning style, and their ability levels will be varied. In these instances, we are giving all students choices of content, process, or product that are at about the same level. These tiers are similar to those in a layer cake—they are all the same size.

The number of groups per tier will vary, as will the number of students per tier. We are not looking to form groups of equal size; we are trying to form groups based on the needs of individual students. For example, tier 1 may have two groups of three students; tier 2 may have five groups of four students; and tier 3 may have one group of two students. When the lesson is tiered by interest or learning style, the same guidelines apply for forming groups—different tiers may have varying numbers of students.

The Differentiated Assessment Component

ASSESSMENT SHOULD BE FORMATIVE, SUMMATIVE, or a combination. The teacher may observe the students as they share their answers during a discussion and jot down notes for a formative assessment of each student or record observations of the various groups on flip cards or sticky notes. For example, which child is struggling with the concept? Which child is moving rapidly and accurately through the material? Whose answers show more thought and insight? Answers to these and other questions will help you determine who needs reteaching and who is ready to go on.

Culminating or summative assessments are given at the end of a unit or large block of study. The teacher could develop a rubric based on a particular product that is developed. The teacher may give a formal paper-and-pencil test with a core block of questions that all students must attempt and a block of questions from which students can select the specific ones they will answer to accumulate the

necessary points. The form of assessment you choose should be based on the lesson design and the purpose of the instruction.

How to Tier a Lesson

TO TAKE A CLOSER LOOK AT THE ANATOMY OF A tiered lesson, we have included a mathematics lesson (shown in **fig. 2**), which was developed for the Indiana Department of Education and field-tested by teachers in the state. When developing a tiered lesson, we found the eight steps described below to be useful.

First, identify the grade level and subject for which you will write the lesson. In this case, the grade level is seventh, and the subject is mathematics.

Second, identify the standard (national, state, district, and so on) that you are targeting. A common mistake for those just beginning to tier is to develop three great activities and then try to force fit them into a tiered lesson. Start with the standard first. If you do not know where you are going, how will you know if you get there? The author of this lesson has selected the Measurement Standard from *Principles and Standards for School Mathematics* (NCTM 2000, pp. 240–47).

Third, identify the key concept and generalization. The key concept follows from the standard. Ask yourself, "What big idea am I targeting?" In this example, students are to work with measurement of perimeter and area. Although many concepts could be covered under the standard that was chosen, this lesson addresses only one. The generalization follows from the concept chosen. Ask yourself, "What do I want the students to know at the end of the lesson, regardless of their placement in the tiers?" In this lesson, all students work with the relationship between the perimeter and area of shapes.

Fourth, be sure students have the background necessary to be successful. What scaffolding is necessary? What must you have already covered or what must the student have already learned? Are there other skills that must be taught first? For this lesson, students should be familiar with the concepts of perimeter and area.

Fifth, determine which part of the lesson (content, process, or product) you will tier. You may choose to tier the *content* (what you want the students to learn), the *process* (the way students make sense out of the content), or the *product* (the outcome at the end of a lesson, lesson set, or unit—often a

Grade: Seventh

Subject: Mathematics

Standard: Measurement

Key Concept: Students work with measurement of perimeter and area.

Generalization: Students see the relationship between perimeter and area of shapes.

Background: This lesson should be presented fairly early in the year. Students should be familiar with the concepts of perimeter and area. The materials needed are multicolored paper squares and hexagons.

Tier 1: Below Grade Level Learners

Pairs of students are given eight squares that are a variety of colors; you could have the students make their own squares from card stock, or you may already have an appropriate manipulative for the students to use. Students are given a worksheet that contains a series of questions about perimeter and area related to the eight squares. Questions about the greatest and least perimeter, greatest and least area, specific areas, and specific perimeters should be included on the worksheet. In addition, students should be asked to draw specific examples for each question.

Tier 2: Grade Level Learners

Pairs of students are given twelve squares that are a variety of colors; you could have the students make their own squares from card stock, or you may already have an appropriate manipulative for the students to use. Students are given a worksheet that contains a series of questions about perimeter and area related to the twelve squares. Questions about the greatest and least perimeter, greatest and least area, specific areas, and specific perimeters should be included on the worksheet. In addition, students should be asked to draw specific examples for each question.

Tier 3: Above Grade Level Learners

Pairs of students are given twelve hexagons that are a variety of colors; you could have the students make their own hexagons from card stock, or you may already have an appropriate manipulative for the students to use. Students are given a worksheet that contains a series of questions about perimeter and area related to the use of the twelve hexagons. Questions about the greatest and least perimeter, greatest and least area, specific areas, and specific perimeters should be included on the worksheet. In addition, students should be asked to draw specific examples for each question.

Assessment: Each worksheet should be graded for accuracy. An extension for each tier would be to ask questions that involve a greater number of their shape or pertain to another shape.

Fig. 2 A lesson tiered in content according to readiness

project). When beginning, we suggest that you tier only one of these three. Once you are comfortable with the process, you might try to tier more than one part in the same lesson. This lesson is tiered in *content*.

Sixth, determine the type of tiering: readiness, interest, or learning style. Readiness is based on the ability levels of the students. Giving a pretest is a good way to assess readiness. *Interest* is based on students' interest in a topic, generally gauged through a survey. *Learning style* may be determined through various learning style inventories. In this lesson, the author chose *readiness*.

Seventh, based on your choices above, determine how many tiers you will need and develop the lesson.

When tiering according to readiness, you may have three tiers: below grade level, at grade level, and above grade level. If you choose to tier in interest or learning style, you may control the number of tiers by limiting choices or using only a few different learning styles. For example, one could choose visual, auditory, and kinesthetic learning preferences. For this lesson, students are placed in one of three tiers based on their ability to work with perimeter and area as assessed by the teacher through observation.

Differentiation means doing something qualitatively different for each tier group. Make sure you keep this in mind when tiering the lessons. Second, be sure that each tier is doing moderately challenging, respectful, and developmentally appropriate

Grade: Sixth

Subject: Mathematics

Standard: Connections

Key Concept: Students use technology, such as computers and calculators, as tools to model patterns.

Generalization: Students create and use mathematical objects to make patterns.

Background: Students have worked with number patterns and have observed patterns of geometric objects. This lesson would most likely be appropriate toward the end of the academic year. Within each tier, the teacher may choose to have the students work in pairs, triads, or quads.

Tier 1: Visual Learners

Groups of students are given a picture from nature, wrapping paper, or other visual item that contains a definite pattern. The students use the item to create the same pattern with numbers using the calculator or a new visual using the computer that conveys the same pattern. Students must write at least one paragraph justifying how and why the new item represents the same pattern found in the original item.

Tier 2: Auditory Learners

Groups of students listen to a musical piece or song that has a definite pattern. The students use the music to create the same pattern with numbers using the calculator or a new visual using the computer that conveys the same pattern. Students must write at least one paragraph justifying how and why the new item represents the same pattern found in the music.

Tier 3: Kinesthetic Learners

Students are taught the steps for the cha-cha or rumba or some other dance that has a definite pattern. The students use the dance to create the same pattern with numbers using the calculator or a new visual using the computer that conveys the same pattern. Students must write at least one paragraph justifying how and why the new item represents the same pattern found in the dance.

Assessment: This lesson is meant to be a fun way to consider patterns and provide extra practice working with numbers, geometric objects, calculators, and computers. Although an assessment is not necessary, you may want to grade the project using a rubric based on students' justifications, ability to work cooperatively, and on originality and creativity.

Fig. 3 A lesson tiered in process according to learning style

work. In other words, no group should be given busy work. We do not want one group doing black-line practice sheets and another doing an exciting activity.

Notice in this lesson that all three tiers are working on perimeter and area. Students in each tier use a specific number of a particular geometric shape to answer a series of questions about perimeter and area. In other words, the content for each tier in the sample lesson, beginning in tier 1 and moving through tier 3, differs from simple to complex and from foundational to transformational, to use Tomlinson's equalizer word pairs (Tomlinson 1999). Tier 1 is the lowest level; tier 3 is the highest level.

Eighth and last, develop the assessment component to the lesson. The assessment can be formative, summative, or a combination of both. Since the standards support both formal and informal assessment, you may record observations of the various groups, develop a rubric for each product that is created, or give a formal paper-and-pencil test

(NCTM 2000, p. 22). Choose the assessment based on your needs and lesson design. A formal assessment, grading each worksheet for accuracy, is used here to determine each student's understanding of perimeter and area.

The second sample lesson, **figure 3**, is tiered in content according to learning style. In this case, students are grouped heterogeneously based on one of three learning preferences: auditory, kinesthetic, or visual. The same eight steps for tiering a lesson apply in this case. In the second lesson, notice that the activities are at the same approximate level of complexity. This would be the "layer cake" model as opposed to the "wedding cake" model used when tiering according to readiness.

Final Thoughts

TIME, ENERGY, AND PATIENCE ARE REQUIRED TO learn to effectively differentiate instruction in an academically diverse classroom. In addition, you

need administrative and peer support, as well as professional development over extended periods of time. Do not expect to have a differentiated classroom by Monday morning. Start small: choose a favorite lesson in your next unit and differentiate it according to the needs of your students. Seek the expertise of specialists such as special and gifted education coordinators, media specialists, and others to collaborate to improve instruction in an academically diverse classroom.

References

- Armstrong, Thomas. *Multiple Intelligences in the Classroom*. Alexandria, VA: ASCD, 1994.
- Gardner, Howard. *Multiple Intelligences: The Theory and Practice*. New York: BasicBooks, 1993.
- Gregory, Gail, and Carolyn Chapman. *Differentiated Instructional Strategies: One Size Doesn't Fit All*. Thousand Oaks, CA: Corwin, 2002.
- Heacox, Diane. *Differentiating Instruction in the Regular Classroom*. Minneapolis: Free Spirit, 2002.
- Martin, Hope. *Multiple Intelligences in the Mathematics Classroom*. Palatine, IL: IRI/SkyLight, 1996.
- National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.
- Smutny, Joan, Sally Walker, and Elizabeth Meckstroth. *Teaching Young Gifted Children in the Regular Classroom*. Minneapolis: Free Spirit, 1997.
- Tomlinson, Carol. *The Differentiated Classroom: Responding to the Needs of All Learners*. Alexandria, VA: ASCD, 1999.
- Winebrenner, Susan. *Teaching Gifted Kids in the Regular Classroom*. Minneapolis: Free Spirit, 1992.

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